Quantum Resonances and Related Topics

Conference in honor of 60th birthday of André Martinez



11 – 13 June 2019 Institut Henri Poincaré (IHP) l'amphithéâtre Hermite

PROGRAM

June 11 (Tuesday)

10:20 – **10:50** Opening

- 10:50 11:40 GRIGIS, Alain (Paris 13) The works of André Martinez
- 11:50 12:40 HELFFER, Bernard (Nantes)
 On the multiplicity of the second eigenvalue of the Laplacian in non simply connected domains–with some numerics–
 (B. Helffer, T. Hoffmann-Ostenhof, F. Jauberteau and C. Léna)

Lunch

- 14:40 15:30 BRIET, Philippe (Toulon) Resonances of metastable molecular systems Work in collaboration with André Martinez
- 15:40 16:30 KLEIN, Markus (Potsdam) Tunneling for a class of difference operators
- **16:30 17:00** Coffee break
- 17:00 17:50 BONY, Jean-François (Bordeaux) Théorie des résonances sans analycité

June 12 (Wednesday)

- 9:30 10:20 SJÖSTRAND, Johannes (Bourgogne) Eigenvalue asymptotics of large Toeplitz matrices with random perturbations
- **10:20 10:50** Coffee break
- 10:50 11:40 ZWORSKI, Maciej (Berkeley) Viscosity limits for 0th order pseudodifferential operators
- 11:50 12:40 GERARD, Christian (Orsay) Analytic wave front set and Quantum Fields on curved spacetimes

Lunch

- 14:40 15:30 ROULEUX, Michel (Toulon) Semi-classical Green functions a joint work with A. Anikin, S. Dobrokhotov and V. Nazaikinskii
- 15:40 16:30 SACCHETTI, Andrea (Modena) Spontaneous symmetry breaking phenomenon in nonlinear Schrödinger equations
- **16:30 17:00** Coffee break
- 17:00 17:50 BRUNEAU, Vincent (Bordeaux) Obstacles for Magnetic Hamiltonians: SSF near Landau levels

Dinner

June 13 (Thursday)

- 9:30 10:20 NAKAMURA, Shu (Gakushuin) Long-range scattering matrix for Schrödinger-type operators
- $10{:}20-10{:}50 \qquad {\rm Coffee \ break}$
- 10:50 11:40 ROBERT, Didier (Nantes) Propagation of wave packets, Herman-Kluk propagators and codimension one crossings
- 11:50 12:40 GRECCHI, Vincenzo (Bologna) Where there are crossings and mutations of resonances

Lunch

14:40 – 15:30 WANG, Xue Ping (Nantes) Gevrey estimates of the resolvent in *N*-body problems

15:40 - 16:30 NIER, Francis (Paris 13)
Persistent cohomology and Arrhenius law, Part II. (joint work with D. Le Peutrec and C. Viterbo)

Scientific committee : HELFFER, B., NAKAMURA, S., SJÖSTRAND, J., ZWORSKI, M. Organizing committee : DUYCKAERTS, T., FUJIIE, S., GRIGIS, A., IVANOVICI, O., RAMOND, T., WATANABE, T.

BONY, Jean-François (Bordeaux)

"Théorie des résonances sans analycité"

Une collaboration avec Laurent Michel et Thierry Ramond.

BRIET, Philippe (Toulon)

"Resonances of metastable molecular systems"

Work in collaboration with André Martinez

We study metastable states associated to a complex resonance ρ describing the predissociation phenomena of a molecular system in the Born-Oppenheimer approximation. We show that when the semiclassical parameter h is small enough, the survival amplitude,

$$\langle e^{-itH}\varphi,\varphi\rangle \sim e^{-it\rho}b(\varphi,h) + r(t,\varphi,h), \quad t>0$$

for a given initial state φ . Where $b(\varphi, h) = 1 + \mathcal{O}(h^{\mu})$ and the remainder term $r(t, \varphi, h) = h^{\nu} \mathcal{O}(t^{-k})$ for some constant $\mu, \nu, k > 0$.

BRUNEAU, Vincent (Bordeaux)

"Obstacles for Magnetic Hamiltonians: SSF near Landau levels"

We consider the 3D Schrödinger operator with constant magnetic field and its perturbations by imposing Dirichlet or Neumann conditions on the boundary of a bounded domain. We discuss the asymptotic behaviour of the associated Krein spectral shift functions (SSF) near the Landau levels. In some cases, we show that these asymptotics involve the logarithmic capacity of the projection of the obstacle onto the plane perpendicular to the magnetic field. It's a joint work with G. Raikov.

GERARD, Christian (Orsay)

"Analytic wave front set and Quantum Fields on curved spacetimes"

GRECCHI, Vincenzo (Bologna)

"Where there are crossings and mutations of resonances"

Usually we (with André) control perturbative resonances to stay isolated from the others and from the top ones varying a complex parameter. In a PT-symmetric double well Hamiltonian the levels are all perturbative resonances, but there are three sets of infinite ones with different stories. Actually in this case the potential is imaginary and the double well can look as a system of a well and barrier, but is not even clear which one is the well and which one the barrier. In case of real levels there are delocalized and bilocalized states as in real double wells. And we prove that for vanishing positive semiclassical parameter infinite pairs of resonances meat and change when the semiclassical localization effect prevails on the symmetry. Quasi real double well potentials still have this effect and could be applied to molecules as other models considered with André.

GRIGIS, Alain (Paris 13)

"The works of André Martinez"

In this talk I will try to give an outline on the works of André Martinez. In particular, I will recall the breakthrough that he made on the Born-Oppenheimer approximation, on the microlocal tunnel effect and on the WKB expansions for systems.

HELFFER, Bernard (Nantes)

"On the multiplicity of the second eigenvalue of the Laplacian in non simply connected domains–with some numerics–"

(B. Helffer, T. Hoffmann-Ostenhof, F. Jauberteau and C. Léna)

We revisit an interesting example proposed by Maria Hoffmann-Ostenhof, Thomas Hoffmann-Ostenhof and Nikolai Nadirashvili of a bounded domain in \mathbb{R}^2 for which the second eigenvalue of the Dirichlet Laplacian has multiplicity 3. We also analyze carefully the first eigenvalues of the Laplacian in the case of the disk with two symmetric cracks placed on a smaller concentric disk in function of their size.

KLEIN, Markus (Potsdam)

"Tunneling for a class of difference operators"

NAKAMURA, Shu (Gakushuin)

"Long-range scattering matrix for Schrödinger-type operators"

For Schrödinger operator-type pseudodifferential operators with longrange perturbations, it is shown that the modified scattering matrix is a Fourier integral operator, and it is a natural quantization of the scattering map for the corresponding modified classical scattering. The spectral properties of scattering matrices for several models are discussed, and it is shown that the scattering matrix can have absolutely continuous spectrum.

NIER, Francis (Paris 13)

"Persistent cohomology and Arrhenius law, Part II."

(joint work with D. Le Peutrec and C. Viterbo)

This work is about the accurate computation of exponentially small eigenvalues of semiclassical Witten Laplacians acting on p-forms. Even when the potential is not a Morse function and possibly Lipschitz (subanalytic) it is possible to prove that the exponential scales are given by persistent (co)homology while the prefactors are related with local possibly solvable models. After stating this result in a simple form, I will explain on examples the spectral versions of the stability theorem of persistence homology.

ROBERT, Didier (Nantes)

"Propagation of wave packets, Herman-Kluk propagators and codimension one crossings"

We construct a Hermann-Kluk type propagator for approximating the solution of a semi-classical system of Schrödinger equations. We consider the case of a data localized along an isolated mode and the situation where this mode intersects smoothly with another one. This relies on the precise study of the propagation of coherent states through codimension one eigenvalue crossings and on Gaussian initial value representation of the initial data.

This is a joint work, in progress, in collaboration with Clotilde Fermanian-Kammerer and Caroline Lasser.

ROULEUX, Michel (Toulon)

"Semi-classical Green functions"

a joint work with A. Anikin¹, S. Dobrokhotov¹ and V. Nazaikinskii¹

Let $H(x,p) \sim H_0(x,p) + hH_1(x,p) + \cdots$ be a semi-classical Hamiltonian on $T^*\mathbf{R}^n$, and $\Sigma_E = \{H_0(x,p) = E\}$ a non critical energy surface. Consider f_h a semi-classical distribution (the "source") microlocalized on a Lagrangian manifold Λ which intersects cleanly the flow-out Λ_+ of the Hamilton vector field X_{H_0} in Σ_E . Using Maslov canonical operator, we look for a semi-classical distribution u_h satisfying the limiting absorption principle and $H^w(x, hD_x)u_h = f_h$ (semi-classical Green function). We focus on the case where the Hamiltonian is of Helmholtz type $H(x,p) = |p|^m \frac{1}{\rho(x)}$, m = 1, 2 and the source is either microlocalized on the conormal of a hypersurface of \mathbf{R}^n , or has the form of a "Bessel beam", i.e. Λ is the "cylinder" $\Lambda = \{x = X(\varphi, \psi) = \varphi \omega(\psi), p = P(\varphi, \psi) = \omega(\psi), \varphi \in \mathbf{R}, \omega \in \mathbf{S}^{n-1}\}$, when n = 2, 3.

SACCHETTI, Andrea (Modena)

"Spontaneous symmetry breaking phenomenon in nonlinear Schrödinger equations"

In this talk we discuss some results for a class of nonlinear models in Quantum Mechanics. In particular we focus our attention to the nonlinear one-dimensional Schrödinger equation with a symmetric double-well potential. In the semiclassical limit we prove that the ground state of the linear model bifurcates when the strength of the nonlinear perturbation assume a critical value, and the kind of bifurcation depends on the nonlinearity power. This line of research is inspired by Grecchi V. and Martinez A., "Non-linear Stark effect and molecular localization", Communications in Mathematical Physics (1995).

¹Ishlinski Institute for Problems of Mechanics, and Moscow Institute of Physics and Technology, Moscow, Russia

References:

- Grecchi V., Martinez A., Sacchetti A., "Destruction of the beating effect for a non-linear Schrödinger equation", Communications in Mathematical Physics (2002).
- Sacchetti A., "Nonlinear double well Schrödinger equations in the semiclassical limit", Journal of Statistical Physics (2005).
- Sacchetti A., "Universal Critical Power for Nonlinear Schrödinger Equations with a Symmetric Double Well Potential", Physical Review Letters (2009).
- Fukuizumi R., Sacchetti A., "Bifurcation and Stability for Nonlinear Schrödinger Equations with Double Well Potential in the Semiclassical Limit", Journal of Statistical Physics (2011).

SJÖSTRAND, Johannes (Bourgogne)

"Eigenvalue asymptotics of large Toeplitz matrices with random perturbations"

This talk is mainly about the most recent one in a series of joint works with Martin Vogel. In earlier works we have considered the asymptotics of eigenvalues of random perturbations of large Jordan matrices and large bidiagonal Toeplitz matrices with a constant entry on each of the "diagonals". We then showed:

- 1) Most of the eigenvalues live near a certain curve determined by the symbol of the matrix and satisfy a Weyl law there.
- 2) A minority of the eigenvalues live away from the symbol image curve and their expected density can be described, even though some intuition behind the formula is still missing.

We review quickly the earlier results and then turn to the case of large Toeplitz matrices with constant non-vanishing entries on each of finitely many "diagonals" (finite band matrices). The main result is that we still have the result 1) above, leaving the problem 2) for future study.

We also hope to present a recent generalization to the case of matrices that are not of finite band type.

WANG, Xue Ping (Nantes)

"Gevrey estimates of the resolvent in *N*-body problems"

In this talk we study the resolvent of N-body Schrödinger operators at the lowest threshold. We show, under some conditions, how to establish a resolvent expansion with Gevrey estimates on the remainder. As application, we give a large-time expansion of solutions to the associated heat equation with subexponential time-decay estimates on the remainder.

ZWORSKI, Maciej (Berkeley)

"Viscosity limits for 0th order pseudodifferential operators"

For self-adjoint pseudodifferential operators of order 0, Colin de Verdière and Saint-Raymond introduced natural dynamical conditions (motivated by the study of internal waves in fluids) guaranteeing absolute continuity of the spectrum. I will present an alternative approach to obtaining their result based on radial propagation estimates from scattering theory (joint work with S. Dyatlov). I will then explain how an adaptation of the Helffer–Sjöstrand theory of scattering resonances shows that in a complex neighbourhood of the continuous spectrum viscosity eigenvalues have limits as viscosity goes to 0. Here the viscosity eigenvalues are the eigenvalues of the original operator to which an anti-self-adjoint elliptic 2nd order operator is added.

Official dinner

There will be an official dinner on Wednesday evening at $7h30\ pm$ at the Restaurant "Au Bistro de la Montagne" .

Address : 38 rue de la Montagne Sainte Geneviéve

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Access to the restaurant :

